Satellite speed

On a circular orbit, with radius \( r \), a satellite has speed (velocity) \( v \):

\[
v = \sqrt{\frac{GM}{r}}
\]

\( G = \) universal gravity constant

\[
G = 3.44 \times 10^{-2} \text{ pound-m}^2/\text{slug}^2
\]

\[
= 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2
\]

\( M = \) mass of Earth

\[
M = 4.94 \times 10^{25} \text{ slugs}
\]

\[
= 5.976 \times 10^{24} \text{ kg}
\]

Example: Typical Space Shuttle mission has main engine cutoff at 200 km. altitude \( r = 6578 \text{ km} \).

\[
v = \sqrt{\frac{3.986 \times 10^5 \text{ km}^3/\text{s}^2}{6578 \text{ km}}}
\]

= 7.8 km/s = 17,400 miles/hour.

At this particular altitude (\( h = 200 \text{ km} \)), if the Shuttle's speed is not exactly 7.7843 km/s, then the orbit will be elliptical.
Orbital Period, $P$

Time for satellite to make one complete trip around an orbit is $P$.

Circular orbit:  \[ P = 2\pi \sqrt{\frac{r^3}{GM}} \]

Elliptical orbit:  \[ P = 2\pi \sqrt{\frac{(r_a + r_p)^3}{8GM}} \]

$r_a = \text{radius of apogee}$

$r_p = \text{radius of perigee}$

Example: Low Earth Orbit (LEO) (circular)

If $r = 6578 \text{ km}$, then period is

\[ P = \sqrt{\frac{4\pi^2 (6578 \text{ km})^3}{3.986 \times 10^5 \text{ km}^3/\text{s}^2}} = 5309 \text{ sec.} \]

\[ = 88 \text{ min.} \]

Note that changing $r$ by 100 km. or so has little effect on the orbital speed or period.
Inclination of an Orbit

Orbits are planar (they lie flat in a geometric plane)

If orbital plane contains Earth's equator, then the orbit has inclination $= 0$.
Otherwise, orbit inclination = angle between equatorial plane and orbital plane
Polar orbit
(inclination = 90°)
Passes over North and South Poles.

Note: Earth turns underneath the orbit, but orbital plane does not rotate! (Newton’s 1st law tells us that the satellite moves in straight line at constant speed unless an unbalanced force deflects it. Gravity does deflect it, making it curve along a circular or elliptical path, but gravity will not make the orbital plane rotate with the Earth!)

Since Earth turns, but orbit does not, eventually the satellite will see all of Earth’s surface — useful for mapping purposes!