Homework #4

Simply-supported, symmetric, specially-orthotropic rectangular plates – Navier method

Consider a simply-supported rectangular plate having the following dimensions:

\[ a = 0.400 \text{ m} \quad b = 0.200 \text{ m} \quad h = \text{TBD m} \]

Consider laminates made from unidirectional AS/3501 laminae, with properties as follows:

\[ h_{\text{ply}} = 0.0001 \text{ m} \]

\[ E_1 = 138 \text{ GPa} \quad E_2 = 9.0 \text{ GPa} \quad G_{12} = 6.9 \text{ GPa} \quad \nu_{12} = 0.30 \quad \rho = 1530 \text{ kg/m}^3 \]

\[ s_1^{(+)} = 1450 \text{ MPa} \quad s_1^{(-)} = 1170 \text{ MPa} \quad s_{2}^{(+)} = 48 \text{ MPa} \quad s_{2}^{(-)} = 248 \text{ MPa} \quad s_{12} = 62.1 \text{ MPa} \]

The plate supports payload electronics having a total mass of 0.25 kg, which you may assume is distributed uniformly over the plate area. Under normal operating conditions, the plate supports lateral loads. The nominal load case is 1 "g" of lateral acceleration (down). The limit load case is defined by a "load factor," \( n = 6 \). The ultimate load is defined by a factor of safety of 1.5.

(a) Lightweight Laminate. Design the lightest balanced, symmetric (possibly cross-ply) laminate you can that can support the limit load without first ply failure, and that has a fundamental vibration frequency higher than 15 Hz. Describe your design rationale.

Laminate Configuration. Use a laminate ply sequence that satisfies the required assumptions. The Navier method we developed assumes symmetry ([B] = 0) and special orthotropy (D16=D26=0). Special orthotropy can be obtained exactly by using a "cross-ply" layup (only 0s and 90s), but can be obtained approximately by using a balanced layup with a large number of (thin) alternating ± layers. If you use such a layup, take D16=D26=0. Calculate the "orthotropy ratio" for the plate (max(D11/D22, D22/D11)).

Static Deflection. Find the lateral deflection of the center of the plate under the limit load condition. How do you think the results would change with orthotropy ratio and aspect ratio?

Static Stress. Plot the variation of the normal stresses (\( \sigma_x \) and \( \sigma_y \)) through the thickness at the center of the plate. Plot the variation of the in-plane shear stress (\( \sigma_{xy} \)) at the (x=0, y=0) corner of the plate, and the transverse shear stresses (\( \sigma_{xz} \) and \( \sigma_{yz} \)) through the thickness at the middle of the (y=0) edge of the plate. Are these the locations where the stresses take on their extreme values? Based on the stresses at the center and corners of the plate, and using the maximum stress criteria, determine the load at which "first-ply failure" is experienced. What is the corresponding factor of safety? What is the failure mode associated with first-ply failure?

(b) Vibration Modes. Determine the plate natural vibration frequencies, for values of \( m \) and \( n \) between 1 and 3. (Note that you may "miss" some modes having frequencies in the range spanned by these.) What is the frequency of the fundamental mode of vibration for the plate, in Hz (cycles/second, not radians/second)?

(c) Elastic Stability. Determine the critical plate buckling load, \( N_0 \) (-\( N_x \)), assuming a load ratio (\( N_y/N_x \)) = 1/4.