This lab is worth 10 points. It has three main objectives:

- to give you a feel for how money grows in an investment and how loans work,
- to give you practice using the discounting and compounding formulas, and
- to give you additional practice using spreadsheets.

1. The following equation gives the future value ($V_n$) of an investment as a function of the initial investment ($V_0$), the interest rate ($i$), and the number of years ($n$) in the investment.

   \[ V_n = (1 + i)^n V_0 \]

In sheet 1 of a new Excel spreadsheet, build a table that gives the future value of an investment for a range of interest rates and time periods according to the following instructions:

- In a pair of cells in the top left part of the spreadsheet, indicate the value of the initial investment. In the left cell, put a label; e.g., “initial investment.” In the right cell, put the value $1,000. I will refer to these cells as the parameter block. The cells should look something like this:

<table>
<thead>
<tr>
<th>Initial investment:</th>
<th>$1,000</th>
</tr>
</thead>
</table>

- In a block below the parameter block create a table that looks something like this:

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

The number of years should go up to 30.

c. I will assume that the $1,000 in the parameter block is in cell B2, that the 0.03 is in B5, and that the number 0 (representing 0 years) is in cell A6. It is not necessary that your numbers are in these exact cells – just be sure to adjust your formula accordingly. Insert the following formula (pay close attention to the dollar signs) into cell B6: =B$2*(1+B$5)^$A6. This gives you the “future value” of $1,000 if invested at 3% for 0 years. Since the time is 0, of course, the “future value” is just $1,000.

d. Now, copy the formula from that cell to all the other empty cells in the table, including the other two currently-empty columns (most likely columns C and D). Note how the cell references change as you copy the formula. The reference to the amount of the initial investment stays fixed. For the references to the number of years, the column stays fixed, but the row changes. For the references to the interest rate, the row stays fixed, but the column
changes. The dollar signs in the formula were specifically placed to work this way. To summarize:

1. A dollar sign before the column reference keeps the column that is referenced fixed, but not the row.
2. A dollar sign before the row reference keeps the row fixed, but not the column.
3. Dollar signs in front of both the row and the column keep the reference fixed on a single cell.

e. Now, create a new table just to the right of the one you created in steps e and d. The new table should have the same column and row headings as the original table. This time, however, use a different formula. In the first row (corresponding to row 6, columns F-H, in my example), no formula will be needed; simply reference the starting amount ($1,000). In the second row (cell F7), enter the following formula: =F6*(1+F$5). This formula multiplies the number above the current cell by one plus the interest rate. Copy this formula into the remaining empty cells in the spreadsheet. This should give you exactly the same numbers as you got in the first table. Explain why. (Type your answer on sheet 2 of your workbook. Label this sheet Answers, and put the names of the members of your group at the top of the sheet. Label sheet 1, with your data for this problem, Prob1.)

f. Now graph the three columns of data in one of your tables. (Use either table.) Use the graph wizard in Excel. Make sure your graph has appropriate axis labels, a title, and a legend. Also, make sure that the x-axis shows the years 0 through 30, in 5-year increments. Include the graph as a separate sheet in your file.

2. The formula for the present value of a finite periodic series is:

\[ V_0 = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1](1+i)^n} \]

where \( R \) is a regular, periodic payment that is received every \( t \) years and ends after a fixed number of years (\( n \)). The future value for this kind of series is given by the following formula:

\[ V_n = \frac{R[(1+i)^n - 1]}{[(1+i)^t - 1]} \]

The idea of this exercise is for you to calculate the present and future values of a periodic series of payments by discounting or compounding each payment individually and then to compare the resulting values to the values you get from the above formulas.

a. In sheet 3 of your spreadsheet (rename this sheet Prob2) create a table like the one shown on the top of the next page. In the Year column (the second column of the table), enter 4 for the first row. For the remaining rows, use a formula that references the value you entered in the first row (the value is 4, but use a reference) and that multiplies the value in the first row by an appropriate amount for that row (i.e., 2 for the second row, 3 for the third row, etc.). The point here is that you should be able to change the value in the first row and have the remaining values adjust accordingly. For the present value column (the fourth column of the table), calculate the present value (corresponding to year 0) of the amount given in the third column. For the future value column (the fifth column), calculate the future value in year 20 of the amount given in the third column. Use formulas in such a way that the formula only
<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
<th>Present Value</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUM</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Check SUM</td>
<td>----</td>
<td>----</td>
<td></td>
</tr>
</tbody>
</table>

Interest Rate: .04

Needs to be entered once for each column and can then be copied down. Use an interest rate of 4%. Calculate the sum of the present value and future value columns in the row labeled SUM.

b. In the row labeled Check SUM, use the formulas given above for the present and future values of a finite periodic series. Your numbers in the SUM and Check SUM rows should match. If so, you have confirmed that the formulas for the present and future values of a finite periodic series actually work. (If not, you most likely have an error somewhere in one of your formulas.) Try some different interest rates and different intervals between payments and make sure all the formulas still work when you change these values.

3. The payment formula for a finite annual series of n payments with a present value of \( V_0 \) is:

\[
R = \frac{V_0 \cdot i \cdot (1 + i)^n}{[(1 + i)^n - 1]}
\]

In other words, this formula can be used to calculate the payment for a loan in the amount of \( V_0 \), at a periodic interest rate of \( i \), and a loan duration of \( n \) periods. You will use it to calculate the payment for a mortgage loan and to look at how much of the payment goes to interest and principal, respectively, over the life of the loan. Since the mortgage is paid monthly, you will need to use a monthly interest rate instead of an annual interest rate.

a. Starting with a new sheet (labeled Prob3), create a parameter block similar to the table at the top of the next page. Use formulas to calculate the equivalent monthly interest rate, the amount financed, the monthly payment for principal and interest, and the total amount paid over the life of the loan. (Note, the numbers in the table below are correct, and you can use them to determine whether you have entered the formulas correctly, but do not just enter the values in the table.) Use the following formula to calculate the equivalent monthly interest rate:

\[
i_m = (1 + i)^{1/12} - 1
\]
Use the payment version of the finite annual series formula (shown above, on page 3) to calculate the monthly payment for principal and interest. Since the payments are monthly, use a monthly interest rate and the number of monthly payments in the place of the annual interest rate and the number of yearly payments. The total amount paid over the life of the loan is just the monthly payment times the number of payments.

b. Now, create a table with a row for each month’s payment and balance information. The table should have a row for each month (0 to 360).
   - Column 1 should show the month number, with values running from 0 to 360.
   - For the remaining columns start by creating column headings and fill in only the values for the first row initially.
     - Column 2 should show the current balance. Reference the amount financed in the parameter block to give the initial value.
     - Column 3 should show the equity value – i.e., the amount of the home’s value actually owned by the homeowner. For row 1 simply reference the down payment.
     - Column 4 should show the interest paid for the month. No payment is made until month 1, so this value should be zero for month 0.
     - Column 5 should show the amount of the payment applied to the principal. Again, since no payment is made until month 1, this value should be zero for month 0.
   - Now, fill in the second row of data. Follow these steps in order.
     - Start with the interest paid. Multiply the balance in Column 2 times the interest rate to get this value.
     - Subtract the amount of interest paid (calculated in Column 4) from the payment (from the parameter block) to obtain the amount of the payment applied to the loan principal.
     - Calculate the current balance by subtracting the amount of the payment applied to the principal from the balance in the previous month.
     - Similarly, the equity value can be calculated by adding the amount of the payment applied to the principal to the equity value in the previous month.
   - Now, copy the second row of formulas down to fill out the rest of the table. The loan balance should equal zero at the end of 360 months.
c. Create two graphs:
   Graph 1: Loan balance and equity.
   Graph 2: Interest paid and amount applied to principal.
   • Use a stacked line graph for each.
   • Put each graph as a new sheet in your spreadsheet.
   • As always, label axes and use legends, titles, etc. appropriately.
   • Make sure your x-axes run from 0 to 360, and set the number of categories between x-axis tick-mark labels to 12.

d. On the Answers sheet, answer the following questions.
   • How much does the monthly payment decrease if the annual interest rate is decreased from 7 to 5 percent?
   • At 7%, what proportion of the first monthly payment goes to interest?
   • At 7%, what proportion of the 100th monthly payment goes to interest?
   • At 7%, what proportion of the total payments over the life of the loan goes to interest?
   • At 5%, what proportion of the total payments over the life of the loan goes to interest?

4. Make sure your Answers sheet lists the names of all of the members of your group who were present today. E-mail your lab spreadsheet to me (mem14@psu.edu) and to Sandor Toth (sft108@psu.edu). The lab is due before class next Tuesday.